## UNIT - IV chapter 1

## RELATIONAL DATABASE DESIGN VIA ER MODELLING

## Relational Database Design Using ER-to-Relational Mapping

Algorithm to convert the basic ER model constructs into relations BASIC ER DIAGRAM

## Figure 9.1

The ER conceptual schema diagram for the COMPANY database.


Fig 2.1
END GOAL: RELATIONAL MODEL

EMPLOYEE


## Fig 2.2

## Step 1: Mapping of Regular Entity Types:

- For each regular (strong) entity type E in the ER schema, create a relation $R$ that includes all the simple attributes of E .
- Include only the simple component attributes of a composite attribute. Choose one of the key attributes of E as primary key for R .
- If the chosen key of E is composite, the set of simple attributes that form it will together form the primary key of R.
- In our example, we create the relations EMPLOYEE, DEPARTMENT, and PROJECT in Figure 2.3 to correspond to the regular entity types EMPLOYEE, DEPARTMENT, and PROJ ECT from Figure 2.1
(a) EMPLOYEE

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

DEPARTMENT

| Dname | Dnumber |
| :--- | :--- |

PROJECT

| Pname | Pnumber | Plocation |
| :--- | :--- | :--- |

## Fig 2.3

## Step 2: Mapping of Weak Entity Types:

- For each weak entity type W in the ER schema with owner entity type E , create a relation R and include all simple attributes of W as attributes of $R$. In addition, include as foreign key attributes of $R$ the primary key Attribute(s) of the relation(s) that correspond to the owner entity type(s): this takes care of the identifying relationship type of W
- In our example, we create the relation DEPENDENT in this step to correspond to the weakbentity type DEPENDENT.
- We include the primary key SSN of the EMPLOYEE relation-which corresponds to the owner entity type-as a foreign key attribute of DEPENDENT
(b) DEPENDENT

| Essn | Dependent_name | Sex | Bdate | Relationship |
| :--- | :--- | :--- | :--- | :--- |

## STEP 3: MAP BINARY 1:1 RELATIONSHIP TYPES:

For each binary $1: 1$ relationship type $R$, identify relation schemas that correspond to entity types participating in $R$

Apply one of three possible approaches:

## Foreign key approach

- Add primary key of one participating relation as foreign key attribute of the other, which will also represent $R$
- If only one side is total, choose it to represent $R$
- Declare foreign key attribute as unique


## Merged relationship approach

- An alternative mapping of a $1: 1$ relationship type is possible by merging the two entity types and the relationship into a single relation.
- This may be appropriate when both participations are total.


## Cross-reference or relationship relation approach

- Create new relation schema for $R$ with two foreign key attributes being copies of both primary keys
- Declare one of the attributes as primary key and the other one as unique
- Add single-valued attributes of relationship type as attributes of $R$


## Step 4: Mapping of Binary 1 :N Relationship Types:

- For each regular binary $1: N$ relationship type $R$, identify the relation $S$ that represents the participating entity type at the $N$-side of the relationship type.
- Include as foreign key in S the primary key of the relation T that represents the other entity type participating in $R$; this is done because each entity instance on the N -side is related to at most one entity instance on the I-side Of the relationship type.
- Include any simple attributes of the I:N relationship type as attributes of S .
- In our example, we now map the I:N relationship types WORKS_FOR, CONTROLS, and SUPERVISION from Figure 2.1. For WORKS_FOR we include the primary key DNUMBER of the DEPARTMENT relation as foreign key in the EMPLOYEE relation and call it DNO.
- For SUPERVISION we include the primary key of the EMPLOYEE relation as foreign key in the EMPLOYEE relation itself because the relationship is recursive-and call it SUPERSSN.
- The CONTROLS relationship is mapped to the foreign key attribute DNUM of PROJECT, which references the primary key DNUMBER of the DEPARTMENT relation.
- An alternative approach we can use here is again the relationship relation option. We create a separate relation R whose attributes are the keys of Sand T, and whose primary key is the same as the key of S


## EMPLOYEE



## Step 5: Mapping of Binary M: N Relationship Types:

- For each binary M:N relationship type R , create a new relation S to represent R.
- Include as foreign key attributes in $S$ the primary keys of the relations that represent the participating entity types; their combination will form the primary key of S .
- Also include any simple attributes of the $\mathrm{M}: \mathrm{N}$ relationship type as attributes of S .
- Notice that we cannot represent an M:N relationship type by a single foreign key attribute in one of the participating relations because of the $\mathrm{M}: \mathrm{N}$ cardinality ratio; we must create a separate relationship relation S .
- In our example, we map the $\mathrm{M}: \mathrm{N}$ relationship type WORKS_ON from Figure 2.1 by creating the relation WORKS_ON in Figure 2.3. We include the primary keys of the PROJECT and EMPLOYEE relations as foreign keys in WORKS_ON and rename them PNO and ESSN, respectively.
- We also include an attribute HOURS in WORKS_ON to represent the HOURS attribute of the relationship type.
- The primary key of the WORKS_ON relation is the combination of the foreign key attributes $\{E S S N$, PNO\}.
(c) WORKS_ON

| Essn | Pno | Hours |
| :--- | :--- | :--- |

## 1. INTRODUCTION TO RELATIONAL MODEL

- The relational model is very simple and elegant: a relational database is a collection of one or more relations, where each relation is a table with rows and columns. This simple tabular representation enables even novice users to understand the contents of a database, and it permits the use of simple, high-level languages to query the data.
- The major advantages of the relational model over the older data models are its simple data representation and the ease with which even complex queries can be expressed.
- The relational model uses Data Definition Language (DDL) and Data Manipulation Language (DML), the standard languages for creating, accessing, manipulation and querying (i.e. retrieving) data in a relational DBMS.
- An important component of a relational model is that it specifies some conditions that must be satisfied while accepting the values from the user which prevents the entry of incorrect information. Such conditions are called as Integrity Constraints (ICs), which enable the DBMS to reject operations that might corrupt the data.


### 1.1 Fundamental Concepts in Relational Data Model

- The main construct for representing data in the relational model is a relation. A relation consists of a relation schema and a relation instance.
- The relation instance is a table, and the relation schema describes the column heads for the table.
- The schema specifies the relation's name, the name of each field (or column, or attribute), and the domain of each field. A domain is referred to in a relation schema by the domain name and has a set of associated values.
- An instance of a relation is a set of tuples, also called records, in which each tuple has the same number of fields as the relation
schema. A relation instance can be thought of as a table in which each tuple is a row, and all rows have the same number of fields. (The term relation instance is often abbreviated to just relation, when there is no confusion with other aspects of a relation such as its schema.)


Table 1.1: An Instance $S 1$ of the Students Relation

- The instance $S 1$ contains six tuples and has, as we expect from the schema, five fields. Note that no two rows are identical. This is a requirement of the relational model-each relation is defined to be a set of unique tuples or rows.


## Domain:

- A domain D is the original sets of atomic values used to model data. By a atomic, we mean that each value in the domain is indivisible as far as the relational model is concerned. A domain is a pool of the values from where one or more attributes ( or columns) can draw their actual values.
- For Examples: The domain of name is the set of character strings that represents names of person.


## Tuple:

- According to the model, every relation or table is made up of many tuples.
- They are also called the records. They are the rows that a table is made up of.

| 53666 | Jones | jones@cs | 18 | 3.4 |
| :--- | :--- | :--- | :--- | :--- |


| 53832 | Guldu | guldu@music | 12 | 2.0 |
| :--- | :--- | :--- | :--- | :--- |

## Relation

- A relation is a subset of the Cartesian product of a list of domains characterized by a name. Given ' $n$ ' domains denoted by D1, D2, ..., Dn, R is a relation defined on the these domains if $\mathrm{R} \leq \mathrm{D} 1$ * D 2 * ... *Dn.
- Relation can be viewed as a "table".
- In that table, each row represents a tuple of data values and each column represents an attribute.


## Attribute

A column of a relation designated by name. The name associated should be meaningful. Each attributes associates with a domain. A relation schema denoted by $R$ is a list of attributes (A1, A2,...., An).

| sid | name | login | age | gpa |
| :---: | :---: | :---: | :---: | :---: |

## The Degree of the Relation

- The degree of the relation is the number of attributes of its relation schema. The degree of the relation of the Table 1.1 is 5 .


## The Cardinality of the Relation

- The cardinality of the relation is the number of tuples in the relation. The cardinality of the relation of the Table 1.1 is 6.


## Characteristic of Relations

- A tuple is a set of values. A relation is a set of tuples. Since a relation is a set, there is no ordering on rows.
- The order of attributes and their values within a relation is not important as long as the correspondence between attributes and values is maintained.
- Each value in a tuple is atomic. That means each value cannot be divided into smaller components. Hence, the composite and multivalued attributes are not allowed in a relation.

Table: The Relation Employee

| DEPT | EID | NAME | SALARY | MOBILE NO |
| :--- | :--- | :--- | :--- | :--- |
| D1 | 20001234 | R.K.Singh | 10000 | 9123456789 |
| D2 | 20001256 | P.Rakesh | 15000 | Null |
| D3 | 20000511 | J.Anitha | 12000 | 9987654321 |
| D4 | 20000210 | Lal <br> Ahmad | 22000 | Null |

The data shows that there are employees with no mobile number because the number might be unknown at this point of time.

## Integrity Constraints over Relations

An integrity constraint (IC) is a condition specified on a database schema and restricts the data that can be stored in an instance of the database. If a database instance satisfies all the integrity constraints specifies on the database schema, it is a legal instance. A DBMS permits only legal instances to be stored in the database.

Many kinds of integrity constraints can be specified in the relational model:

## Domain Constraints:

A relation schema specifies the domain of each field in the relation instance. These domain constraints in the schema specify the condition that each instance of the relation has to satisfy: The values that appear in a column must be drawn from the domain associated with that column. Thus, the domain of a field is essentially the type of that field.

## Key Constraints:

A Key Constraint is a statement that a certain minimal subset of the fields of a relation is a unique identifier for a tuple.

## Super Key:

An attribute or set of attributes that uniquely identifies a tuple within a relation.

However, a super key may contain additional attributes that are not necessary for a unique identification.
Ex: The customer_id of the relation customer is sufficient to distinguish one tuple from other. Thus, customer_id is a super key. Similarly, the
combination of customer_id and customer_name is a super key for the relation customer. Here the customer_name is not a super key, because several people may have the same name.

We are often interested in super keys for which no proper subset is a super key. Such minimal super keys are called candidate keys.

## Candidate Key:

A super key such that no proper subset is a super key within the relation.
There are two parts of the candidate key definition:
i. Two distinct tuples in a legal instance cannot have identical values in all the fields of a key.
ii. No subset of the set of fields in a candidate key is a unique identifier for a tuple.

A relation may have several candidate keys.
Ex: The combination of customer_name and customer_street is sufficient to distinguish the members of the customer relation. Then both, \{customer_id\} and \{customer_name, customer_street\} are candidate keys. Although customer_id and customer_name together can distinguish customer tuples, their combination does not form a candidate key, since the customer_id alone is a candidate key.

## Primary Key:

The candidate key that is selected to identify tuples uniquely within the relation. Out of all the available candidate keys, a database designer can identify a primary key. The candidate keys that are not selected as the primary key are called as alternate keys.

## Features of the primary key:

i. Primary key will not allow duplicate values.
ii. Primary key will not allow null values.
iii. Only one primary key is allowed per table.

Ex: For the student relation, we can choose student_id as the primary key.

## Foreign Key:

Foreign keys represent the relationships between tables. A foreign key is a column (or a group of columns) whose values are derived from the primary key of some other table.

The table in which foreign key is defined is called a Foreign table or Details table. The table that defines the primary key and is referenced by the foreign key is called the Primary table or Master table.

## Features of foreign key:

Records cannot be inserted into a detail table if corresponding records in the master table do not exist.

Records of the master table cannot be deleted or updated if corresponding records in the detail table actually exist.

## General Constraints:

Domain, primary key, and foreign key constraints are considered to be a fundamental part of the relational data model. Sometimes, however, it is necessary to specify more general constraints.

For example, we may require that student ages be within a certain range of values. Giving such an IC, the DBMS rejects inserts and updates that violate the constraint.

Current database systems support such general constraints in the form of table constraints and assertions. Table constraints are associated with a single table and checked whenever that table is modified. In contrast, assertions involve several tables and are checked whenever any of these tables is

## INTRODUCTION TO SCHEMA REFINEMENT

Conceptual database design gives us a set of relation schemas and integrity constraints (ICs) that can be regarded as a good starting point for the final database design. This initial design must be refined by taking the ICs into account and also by considering performance criteria and typical workloads.

A major aim of relational database design is to minimize data redundancy. The problems associated with data redundancy are illustrated as follows:

## Problems caused by redundancy

Storing the same information in more than one place within a database is called redundancy and can lead to several problems:

- Redundant Storage: Some information is stored repeatedly.
- Update Anomalies: If one copy of such repeated data is updated, an inconsistency is created unless all copies are similarly updated.
- Insertion Anomalies: It may not be possible to store certain information unless some other, unrelated, information is stored as well.
- Deletion Anomalies: It may not be possible to delete certain information without losing some other, unrelated, information as well.

Ex: Consider a relation, Hourly_Emps(ssn, name, lot, rating, hourly_wages, hours_worked)

The key for Hourly_Emps is ssn. In addition, suppose that the hourly_wages attribute is determined by the rating attribute. That is, for a given rating value, there is only one permissible hourly_wages value. This IC is an example of a functional dependency. It leads to possible redundancy in the relation Hourly_Emps, as shown below:

| ssn | name | lot | rating | hourly_wages | hours_worked |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 567 | Adithya | 48 | 8 | 10 | 40 |
| 576 | Devesh | 22 | 8 | 10 | 30 |
| 574 | Ayush <br> Soni | 35 | 5 | 7 | 30 |
| 597 | Rajasekhar | 35 | 5 | 7 | 32 |
| $5 c 1$ | Sunil | 35 | 8 | 10 | 40 |

If the same value appears in the rating column of two tuples, the IC tells us that the same value must appear in the hourly_wages column as well. This redundancy has the following problems:

- Redundant Storage: The rating value 8 corresponds to the hourly_wage 10, and this association is repeated three times.
- Update Anomalies: The hourly_wage in the first tuple could be updated without making a similar change in the second tuple.
- Insertion Anomalies: We cannot insert a tuple for an employee unless we know the hourly_wage for the employee's rating value.
- Deletion Anomalies: If we delete all tuples with a given rating value (e.g., we delete the tuples for Ayush Soni and Rajasekhar) we lose the association between that rating value and its hourly_wage value.


## Null Values

Null values cannot provide a complete solution, but they can provide some help.

Consider the example Hourly_Emps relation. Here null values cannot help to eliminate redundant storage, update or deletion anomalies. It appears that they can address insertion anomalies. For instance, we can insert an employee tuple with null values in the hourly wage field. However, null values cannot address all insertion anomalies. Thus, null values do not provide a general solution to the problems of redundancy, even though they can help in some cases.

## Decompositions

A decomposition of relation schema $\boldsymbol{R}$ consists of replacing the relation schema by two or more relation schemas that each contain a subset of the attributes of $R$ and together include all attributes in $R$.

Ex: We can decompose Hourly_Emps into two relations:
Hourly_Emps2(ssn, name, lot, rating, hours_worked)
Wages(rating, hourly_wages)

## Problems Related to Decomposition

Two important questions must be asked during decomposition process:

1. Do we need to decompose a relation?
2. What problems does a given decomposition cause?

To answer a first question, several normal forms have been proposed for relations. If a relation schema is in one of these normal forms, we know that certain kinds of problems cannot arise.

With respect second question, two properties of decomposition are to be considered:

The lossless-join property enables us to recover any instance of the relation of the decomposed relation from corresponding instances of the smaller relations.

The dependency-preservation property enables us to enforce any constraint on the original relation by simply enforcing some constraints on each of the smaller relations. That is, we need not perform joins of the smaller relations to check whether a constraint on the original relation is violated.

## Functional Dependencies

A functional dependency (FD) is a kind of IC that generalizes the concept of a key.

Let R be a relation schema and let X and Y be nonempty sets of attributes in $R$. We say that an instance $r$ of $R$ satisfies the $F D X \rightarrow Y^{1}$ if the following holds for every pair of tuples $t 1$ and $t 2$ in $r$ :

If $\mathrm{t} 1 . \mathrm{X}=\mathrm{t} 2 . \mathrm{X}$, then $\mathrm{t} 1 . \mathrm{Y}=\mathrm{t} 2 . \mathrm{Y}$.
An $\mathrm{FD} \mathrm{X} \rightarrow \mathrm{Y}$ says that if two tuples agree on the values in attributes X , they must also agree on the values in attributes Y.

Ex: The FD $\mathrm{AB} \rightarrow \mathrm{C}$ is satisfied by the following instance:


[^0]| a1 | b1 | c1 | d1 |
| :--- | :--- | :--- | :--- |
| a1 | b1 | c1 | d2 |
| a1 | b2 | c2 | d1 |
| a2 | b1 | c3 | d1 |

Here, if we add a tuple <a1, b1, c2, d1> to the instance shown in figure, the resulting instance would violate the FD.

## Reasoning about FDs

Given a set of FDs over a relation schema R, typically several additional FDs hold over R whenever all of the given FDs hold.

Ex: Consider Workers(ssn, name, lot, did, since)

With given FDs $s s n \rightarrow$ did and did $\rightarrow$ lot. Then, in any legal instance of Workers, if two tuples have the same ssn value, they must have the same did value, and because they have the same did value, they must also have the same lot value. Therefore, the FD ssn $\rightarrow$ lot also holds on Workers.

## Closure of a Set of FDs

The set of all FDs implied by a given set $F$ of FDs is called the closure of $\mathbf{F}$, denoted as $F+$. The closure $F+$ can be calculated by using the following Armstrong's Axioms rules. Let $\mathrm{X}, \mathrm{Y}$, and Z be the sets of attributes over a relation schema R:

- Reflexivity: If X is a super set of Y , then $\mathrm{X} \rightarrow \mathrm{Y}$.
- Augmentation: If $\mathrm{X} \rightarrow \mathrm{Y}$, then $\mathrm{XZ} \rightarrow \mathrm{YZ}$ for any Z .
- Transitivity: If $\mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{Y} \rightarrow Z$, then $\mathrm{X} \rightarrow Z$.
- Union: If $\mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{X} \rightarrow \mathrm{Z}$, then $\mathrm{X} \rightarrow \mathrm{YZ}$.
- Decomposition: If $\mathrm{X} \rightarrow \mathrm{YZ}$, then $\mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{X} \rightarrow Z$.


## Ex1:

Consider a relation schema ABC with $\mathrm{FDs} \mathrm{A} \rightarrow \mathrm{B}$ and $\mathrm{B} \rightarrow \mathrm{C}$.
From transitivity, we get $\mathrm{A} \rightarrow \mathrm{C}$.
From augmentation, we get $\mathrm{AC} \rightarrow \mathrm{BC}, \mathrm{AB} \rightarrow \mathrm{AC}, \mathrm{AB} \rightarrow \mathrm{CB}$.

## Ex2:

Contracts(contractid, supplierid, projectid, deptid, partid, qty, value)
We denote the schema for Contracts as CSJDPQV.
The following are the given FDs:
i) $\quad C \rightarrow C S J D P Q V$.
ii) $J P \rightarrow C$.
iii) $S D \rightarrow P$.

Several additional FDs hold in the closure of the set of given FDs:

From $J P \rightarrow C$ and $C \rightarrow C S J D P Q V$, and transitivity, we infer $J P \rightarrow C S J D P Q V$.

From $S D \rightarrow P$ and augmentation, we infer $S D J \rightarrow J P$.
From $S D J \rightarrow J P, J P \rightarrow C S J D P Q V$, and transitivity, we infer $S D J \rightarrow C S J D P Q V$.

## Note:

- In a trivial FD, the right side contains only attributes that also appear on the left side. Using reflexivity, we can generate all trivial dependencies, which are of the form:
$X \rightarrow Y$, where $Y$ is a subset of $X, X$ is a subset of $A B C$, and $Y$ is a subset of $A B C$.

From augmentation, we get the nontrivial dependencies.

## Attribute Closure

If we want to check whether a given dependency, say, $\mathrm{X} \rightarrow \mathrm{Y}$, is in the closure $F+$, we can do so efficiently without computing $F+$. We first compute the attribute closure $\mathbf{X}+$ with respect to $F$, which is the set of attributes A such that $\mathrm{X} \rightarrow \mathrm{A}$ can be inferred using Armstrong Axioms. The algorithm for computing the attribute closure of a set X of attributes is shown below:
closure $=X$;
repeat until there is no change: $\{$
if there is an $\mathrm{FD} U \rightarrow V$ in $F$ such that $U \in$ closure, then set closure $=$ closure U V
\}

## CHARACTERISTICS OF FUNCTIONAL DEPENDENCY

(1) It deals with the 1-1 Relationship between attributes and rearly it will also talk about $1-\mathrm{M}$
(2) F.D must be defined on the scheme but not instances.
(3) F.D must be a Non-trivial.
(4) In trivial F.D RHS is a complete subset of LHS Eg: ABC $\rightarrow \mathrm{BC}$
(5) In non-trivial F.D at least one of the RHS attributes is not a subset of LHS Eg: ABC $\rightarrow$ BD
(6) In a complete non-trivial F.D none of the RHS attributes are the subset of LHS Eg: $\mathrm{ABC} \rightarrow \mathrm{DE}$

Once the F.D's are identified from semantics then additional F.D's can be derived from the existing set.

Eg: $F_{1}$ from semantics and $F_{2}$ from $F_{1}$ then total $F$.D's $=F_{1}+F_{2}$.The input for the normalization process should be $\mathrm{F}_{1}+\mathrm{F}_{2} . \mathrm{F}_{2}$ can be identified in the different ways.
(1) By using interference rules.
(2) By closure set of attributes.

## INFERENCE RULES

(1) Reflexive: If ' $B$ ' is a subset of ' $A$ ' then always ' $A$ ' can determine ' $B$ ' $\mathrm{A} \rightarrow \mathrm{B}$
(2) Augmentation: If $A \rightarrow B$ then $A C \rightarrow B C$
(3) Transitive: If $A \rightarrow B$ and $B \rightarrow C$ then $A \rightarrow C$
(4) Union: It is applied for the LHS attributes i.e., If $A \rightarrow B, A \rightarrow C$ then $\mathrm{A} \rightarrow \mathrm{BC}$
(5) Decomposition: If $A \rightarrow B C$ then we can write it as $A \rightarrow B$ and $A \rightarrow C$
(6) Composition: If $\mathrm{A} \rightarrow \mathrm{B}$ and $\mathrm{C} \rightarrow \mathrm{D}$ then, $\mathrm{AC} \rightarrow \mathrm{BD}$
(7) Self determination: $A \rightarrow A, B \rightarrow B$
$1 \mathbf{Q}$ Find the additional F.D's derived from $\mathrm{F}_{1}$ where a set of F.D's from semantics.
$\left.\begin{array}{l}\text { (1) } \mathrm{A} \rightarrow \mathrm{B} \\ \text { (2) } \mathrm{B} \rightarrow \mathrm{C}\end{array}\right\} \mathrm{A} \rightarrow \mathrm{C}$
$\left.\begin{array}{l}\text { (3) } \mathrm{C} \rightarrow \mathrm{D} \\ \text { (4) } \mathrm{D} \rightarrow \mathrm{E} \\ \text { (5) } \mathrm{D} \rightarrow \mathrm{H}\end{array}\right\} \mathrm{D} \rightarrow \mathrm{EH}$
$\left.\begin{array}{l}\text { (6) } \mathrm{E} \rightarrow \mathrm{F} \\ \text { (7) } \mathrm{F} \rightarrow \mathrm{G} \\ \text { (8) } \mathrm{G} \rightarrow \mathrm{H}\end{array}\right\} \mathrm{F} \rightarrow \mathrm{H}$ $\begin{array}{r}\mathrm{A} \rightarrow \mathrm{C} \\ \mathrm{F}_{2}=\mathrm{D} \rightarrow \mathrm{EH} \\ \mathrm{F} \rightarrow \mathrm{H}\end{array}$
By transitive rule, $\mathrm{A} \rightarrow \mathrm{C}$ from (1) \& (2)
By union rule, $\mathrm{D} \rightarrow \mathrm{EH}$ from (4) \& (5)
By transitive rule, $\mathrm{F} \rightarrow \mathrm{H}$ from (7) \& (8)

## CLOSURE SET OF ATTRIBUTES

## Algorithm used to identify the closure set of attributes:

(1) Let ' X ' be a set of attributes that will become the closure.
(2) Repeatedly search for a F.D where the LHS of F.D is a part of ' $X$ ' then add RHS of the F.D to ' X ' is already not available.
(3) Repeat step (2) as many times as necessary until no more attributes can be added to ' X '.
(4) The set ' $X$ ' after no more attributes can be added to ' $X$ ' will become a closure set.

## Applications of closure set of attributes:

(1) To identify the additional F.D's.
(2) To identify the keys.
(3) To identify the equivalences of the F.D's
(4) To identify irreducible set of F.D's or canonical forms of F.D's or standard form of F.D's.

## $2 Q$ Consider a relation ABCDEFG and FDs are

$\mathrm{A} \rightarrow \mathrm{B}$
$\mathrm{BC} \rightarrow \mathrm{DE}$
$\mathrm{AEG} \rightarrow \mathrm{G}$

Find $\mathrm{AC}^{+}$

Ans: $X=A C$
$=\mathrm{ACB}$
$=\mathrm{ABCDE}$
$\mathrm{AC}^{+}=\mathrm{ABCDE}$

## 3Q For the relation $\mathbf{R}(A B C D E)$ and FDs are

$\mathrm{A} \rightarrow \mathrm{BC}$
$\mathrm{CD} \rightarrow \mathrm{E}$
$\mathrm{B} \rightarrow \mathrm{D}$
$\mathrm{E} \rightarrow \mathrm{A}$
Find $\mathrm{B}^{+}, \mathrm{AB}^{+} \& \mathrm{CD}^{+}$

Ans: $\mathrm{X}=\mathrm{B}$

$$
=B D
$$

$$
\mathrm{B}^{+}=\mathrm{BD}
$$

Find $\mathrm{AB}^{+}$

$$
\begin{gathered}
\mathrm{X}=\mathrm{AB} \\
=\mathrm{ABC} \\
=\mathrm{ABCD} \\
=\mathrm{ABCDE} \\
\mathrm{AB}^{+}=\mathrm{ABCDE}
\end{gathered}
$$

Find $\mathrm{CD}^{+}$

$$
\begin{gathered}
\mathrm{X} \quad=\mathrm{CD} \\
=\mathrm{CDE} \\
=\mathrm{ACDE} \\
=\mathrm{ABCDE} \\
\mathrm{CD}^{+}=\mathrm{ABCDE}
\end{gathered}
$$

$4 Q$ For a relation $\mathrm{R}(\mathrm{ABCDEF})$ and FDs are
$\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{BC} \rightarrow \mathrm{AD}$
$\mathrm{D} \rightarrow \mathrm{E}$
$\mathrm{CF} \rightarrow \mathrm{B}$
Find $\mathrm{AB}^{+}$

Ans: Find $\mathrm{AB}^{+}$

$$
\begin{aligned}
& \mathrm{X}=\mathrm{AB} \\
& =\mathrm{ABC} \\
& =\mathrm{ABCD} \\
& =\mathrm{ABCDE}
\end{aligned}
$$

$$
\mathrm{AB}^{+}=\mathrm{ABCDE}
$$

## (1) Identifying the additional F.D's

To check any F.D's like $\mathrm{A} \rightarrow \mathrm{B}$ can be determined from $\mathrm{F}_{1}$ or not.

Complete $\mathrm{A}^{+}$from $\mathrm{F}_{1}$ is $\mathrm{A}^{+}$includes B also then, $\mathrm{A} \rightarrow \mathrm{B}$ can be derived as a F.D in $\mathrm{F}_{2}$.

Q5 Check $\mathrm{D} \rightarrow$ Acan be derived from $\mathrm{F}_{1}$
$\mathrm{F}_{1}$ :

$$
\begin{aligned}
& \mathrm{AB} \rightarrow \mathrm{C} \\
& \mathrm{BC} \rightarrow \mathrm{AD} \\
& \mathrm{D} \rightarrow \mathrm{E} \\
& \mathrm{CF} \rightarrow \mathrm{~B}
\end{aligned}
$$

i. $\quad \mathrm{D} \rightarrow \mathrm{A}$

$$
\begin{aligned}
\mathrm{X} & =\mathrm{D} \\
& =\mathrm{DE} \\
& \therefore \mathrm{D}^{+}=\mathrm{DE}
\end{aligned}
$$

$\therefore$ D $\rightarrow$ A i.e. Cannot be determine A
ii. $\quad \mathrm{AB} \rightarrow \mathrm{D}$

$$
\begin{aligned}
\mathrm{X} & =\mathrm{AB} \\
& =\mathrm{ABC} \\
& =\mathrm{ABCD} \\
& \therefore \mathrm{AB}^{+}=\mathrm{ABCD}
\end{aligned}
$$

iii. $\mathrm{AB} \rightarrow \mathrm{F}$

$$
\begin{aligned}
& \mathrm{X}=\mathrm{AB} \\
& \quad=\mathrm{ABC} \\
&=\mathrm{ABCD} \\
&=\mathrm{ABCDE} \\
& \therefore \mathrm{AB}^{+}=\mathrm{ABCDE}
\end{aligned}
$$

$\therefore \mathrm{AB} \rightarrow \mathrm{F}$ cannot be determine F

Q6 Find BCD $\rightarrow \mathrm{H}$ can be derived from $\mathrm{F}_{1}$

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{BC} \\
& \mathrm{CD} \rightarrow \mathrm{E} \\
\mathrm{~F}_{1:}: & \mathrm{E} \rightarrow \mathrm{C} \\
& \mathrm{D} \rightarrow \mathrm{AEH} \\
& \mathrm{AEH} \rightarrow \mathrm{BD} \\
& \mathrm{DH} \rightarrow \mathrm{BC}
\end{aligned}
$$

## Sol:

i. $\quad \mathrm{X}=\mathrm{BCD}$

$$
=\mathrm{BCDE}
$$

$$
=\mathrm{BCDEH}
$$

$$
=\mathrm{BCDEAH}
$$

$$
\therefore \mathrm{BCD}^{+}=\mathrm{BCDEAH}
$$

$$
\therefore \mathrm{BCD} \rightarrow \mathrm{H}
$$

$$
\text { (ii) } \begin{gathered}
\quad \mathrm{ABC} \rightarrow \mathrm{H} \\
\mathrm{X}=\mathrm{ABC} \\
\therefore \mathrm{ABC} \rightarrow \mathrm{H}
\end{gathered}
$$

## (2) Identification of key by using closure set as attributes

(i) A primary key
(ii) Composite primary ey
(iii)Candidate keys
(iv) Foreign keys
(v) Surrogate key
(vi) Super key

Max.no.of Foreign keys=1024.

A key attribute: An attribute that is capable of identifying all other attributes in a given table.

## (i) Primary key:

It is an unique value attribute in a table to enforce entity integrity and ti identify rows in the table uniquely.

## (ii) Composite Primary Key:

Sometimes single attribute is not sufficient to identify uniquely the rows in the table so, we combine 2 or more attributes to identify the rows uniquely.

## (iii) Candidate keys:

Sometimes 2 or more independent attribute or attributes can be used to identify the rows uniquely Eg :( vech no,veng no, purchase date) Either vehicle no or vehicle engine no can be used as a key attribute then they are called as candidate keys one of the candidate key can be elected as primary key.

## (iv) Surrogate key:

Sometimes even if you combine all the attributes in the table they may not have unique values.
$\therefore$ To identify the rows uniquely we will use a system generated key called as surrogate key.

## (v) Foreign key:

It is used to enforce referential integrity and an attribute in a table can be called as a foreign key attribute that refer primary key in same or different table.

## Q1 If R(ABCDEH) and FDs are

$\mathrm{A} \rightarrow \mathrm{BC}$
$\mathrm{CD} \rightarrow \mathrm{E}$
$\mathrm{E} \rightarrow \mathrm{C}$
$\mathrm{D} \rightarrow \mathrm{AEH}$
$\mathrm{AEH} \rightarrow \mathrm{BD}$
$\mathrm{DH} \rightarrow \mathrm{BC}$

## Find keys?

Super key=ABCDEH
Now find $\mathrm{A}^{+}=\mathrm{ABC}$
$\mathrm{E}^{+}=\mathrm{EC}$
$\mathrm{D}^{+}=\mathrm{DAEH}$
=ABCDEH
$\therefore$ D is key
$>$ If the closure of any of the LHS attributes are combinations of the LHS attributes includes all the attributes in the table then that will become a key in the table.
$>$ A table can have 2 or more keys.

## Q2 Consider a relation with five attributes ABCDE and FDs are

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~B} \\
& \mathrm{BC} \rightarrow \mathrm{E} \\
& \mathrm{ED} \rightarrow \mathrm{~A}
\end{aligned}
$$

## List all keys for R.

Sol: Super key: ABCDE

| $\mathrm{A}^{+}=\mathrm{AB}$ | X |  |
| :--- | :--- | :--- |
| $\mathrm{BC}^{+}=\mathrm{BCE}$ | X |  |
| $\mathrm{ED}^{+}=\mathrm{ABDE}$ | X |  |
| $\mathrm{AB}^{+}=\mathrm{AB}$ |  | X |
| $\mathrm{AC}^{+}=\mathrm{ABCE}$ | X |  |
| $\mathrm{BD}^{+}=\mathrm{BD}$ | X |  |
| $\mathrm{ABC}^{+}=\mathrm{ABCE}$ |  |  |
| $\mathrm{BCD}^{+}=\mathrm{BCDEA}$ | $\sqrt{ }$ |  |
| $\mathrm{ACD}^{+}=\mathrm{ABCDE}$ | $\sqrt{ }$ |  |
| $\mathrm{CDE}^{+}=\mathrm{ADEBC}$ | $\sqrt{ }$ |  |

Q3 $\mathbf{R}(\mathrm{ABCDE}) \& \mathrm{FDs}$ are

$$
\mathrm{AB} \rightarrow \mathrm{C}
$$

$$
\mathrm{CD} \rightarrow \mathrm{E}
$$

$$
\mathrm{DE} \rightarrow \mathrm{~B}
$$

## List all keys for $R$.

Sol: Super key=ABCDE
$\mathrm{AB}^{+}=\mathrm{ABC}$
$\mathrm{CD}^{+}=\mathrm{CDE}$
$\mathrm{DE}^{+}=\mathrm{DEB}$
$\mathrm{ABC}^{+}=\mathrm{ABC}$
$\mathrm{ABD}^{+}=\mathrm{ABDCE}$
$\mathrm{ABE}^{+}=\mathrm{ABEC}$
$\mathrm{ACD}^{+}=\mathrm{ACDEB}$
ABD \& ACD are keys

## Q4 R(ABCDEGGHIJ) \&FDs are

$\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{A} \rightarrow \mathrm{DE}$
$\mathrm{B} \rightarrow \mathrm{F}$
$\mathrm{F} \rightarrow \mathrm{GH}$
$\mathrm{D} \rightarrow \mathrm{RJ}$

## List all keys for $\mathbf{R}$

## Note:

Sometimes all the attributes in the table may not appear in F.D's

Eg: D $\rightarrow$ I may be available instead of $\mathrm{D} \rightarrow \mathrm{IJ}$
$\therefore \mathrm{AB}^{+}=\mathrm{ABCDEFGHIJ}$
$\therefore$ The key for the relation $\mathrm{R}=\mathrm{ABJ}$ the missing attributes from the F.D's must be attached to the closure.

Q5 R(ABCDEFGHIJ) \& FDs are
$A B \rightarrow C$
$B D \rightarrow E F$
$A D \rightarrow G H$
$A \rightarrow I$
$H \rightarrow J$

## List all keys for $\mathbf{R}$

## Sol:

Super key=ABDH

| $\mathrm{AB}^{+}=\mathrm{ABC}$ |  | X |
| :--- | :---: | :---: |
| $\mathrm{BD}^{+}=\mathrm{BDEF}$ |  | X |
| $\mathrm{AD}^{+}=\mathrm{ADGHIJ}$ | X |  |
| $\mathrm{ABD}^{+}=\mathrm{ABDCEFGHIJ}$ | $\sqrt{ }$ |  |
| $\mathrm{ABH}^{+}=\mathrm{ABHCIJ}$ | X |  |
| $\therefore \mathrm{ABD}^{+}=\mathrm{ABCDEFGHIJ}$ |  |  |

## (3) To identify equivalence of F.D

Different database designers may define different F.D's sets from the same requirements.To evaluate whether they are equivalent if we are able to derive all F.D's in $G$ from $F$ and vice-versa.

## Q1 Consider the following two sets of FDs

$$
\begin{gathered}
\mathrm{F}=\begin{array}{l}
\mathrm{A} \rightarrow \mathrm{C} \\
\mathrm{AC} \rightarrow \mathrm{D} \\
\mathrm{E} \rightarrow \mathrm{AD} \\
\mathrm{E} \rightarrow \mathrm{H}
\end{array} \\
\mathrm{G}=\begin{array}{l}
\mathrm{A} \rightarrow \mathrm{CD} \\
\mathrm{E} \rightarrow \mathrm{AH}
\end{array}
\end{gathered}
$$

Find the equivalence of two sets of FDs.
Step 1: Take set F and enclose all FD's in G that can be derived from F.

$$
\mathrm{A} \rightarrow \mathrm{CD}
$$

$\mathrm{A}^{+}$from F

$$
\begin{aligned}
& \quad \mathrm{X}=\mathrm{A} \\
&=\mathrm{AC} \\
&=\mathrm{ACD} \\
& \therefore \mathrm{~A} \rightarrow \mathrm{CD} \text { can be derived from } \mathrm{F} \\
& \mathrm{E} \rightarrow \mathrm{AH}
\end{aligned}
$$

$\mathrm{E}^{+}$from F

$$
\begin{aligned}
& \quad \mathrm{X}=\mathrm{E} \\
& =\mathrm{EAD} \\
& =\mathrm{EADH} \\
& \therefore \mathrm{E} \rightarrow \mathrm{AH} \text { can be derived from } \mathrm{F}
\end{aligned}
$$

Step 2: Take set G and enclose all F.D's in F that can be derived from G.

$$
\mathrm{A} \rightarrow \mathrm{C}
$$

$\mathrm{A}^{+}$from G

$$
\begin{aligned}
& \mathrm{X}=\mathrm{A} \\
& =\mathrm{ACD}
\end{aligned}
$$

$\therefore \mathrm{A} \rightarrow$ C can be derived from G

$$
\begin{aligned}
\mathrm{X} & =\mathrm{AC} \\
& =\mathrm{ACD}
\end{aligned}
$$

$\mathrm{E} \rightarrow \mathrm{AD}$
$\mathrm{X}=\mathrm{E}$

$$
=\mathrm{EAH}
$$

$$
=\mathrm{EAHCD}
$$

$\therefore E \rightarrow A H \& E \rightarrow A D$ from $G$
$\therefore \mathrm{F} \equiv \mathrm{G}$ so, G is preferable as it contains less FDs .

## Q2 Consider two sets of FDs on the attributes ABCDE

$$
\begin{aligned}
\mathrm{F}= & \mathrm{B} \rightarrow \mathrm{CD} \\
& \mathrm{AD} \rightarrow \mathrm{E} \\
& \mathrm{~B} \rightarrow \mathrm{~A} \\
& \mathrm{~B} \rightarrow \mathrm{CDE} \\
\mathrm{G}= & \mathrm{B} \rightarrow \mathrm{ABC} \\
& \mathrm{AD} \rightarrow \mathrm{E}
\end{aligned}
$$

## Find whether they are equivalent or not

## Sol:

Step 1:
$\mathrm{B} \rightarrow \mathrm{CDE}$
$\mathrm{B}^{+}$from F

$$
\begin{aligned}
& \mathrm{X}=\mathrm{B} \\
= & \mathrm{BCDA} \\
= & \mathrm{ABCDE}
\end{aligned}
$$

$\therefore$ All FD's are derivable from F .

## Step 2:

$B \rightarrow C D$
$B^{+}$from $G$

$$
\begin{aligned}
& \mathrm{X}=\mathrm{B} \\
= & \mathrm{BCDE} \\
= & \mathrm{ABCDE}
\end{aligned}
$$

$\therefore$ All FD's are derivable from G.
$\therefore \mathrm{F} \equiv \mathrm{G}$
F is preferable
$\therefore$ No of dependencies are less.

## (4) To identify the irreducible form of FD's / canonical Form

Once $F_{1}$ is identified from the semantics and $F_{2}$ is derived from $F_{1}$ we get total F.D's i.e F but before making a move to the normalization process with $\mathrm{F}, \mathrm{F}$ must be evaluated for redundant attribute on the LHS and RHS of F.D's and it is a four step process.

Step 1: Have single attributes on the RHS for every FD.
Step 2: Evaluate all F.D's in step 1 for their necessity. If they are not necessary, remove them from the list.

Step 3: Evaluate the necessity of the RHS attributes in FD's obtained from step 2.If they are not necessary remove from FD.

Step 4: Apply the union rule for common to LHS attribute in the FD's obtained from step 3.Then we will get irreducible set.

## Q1 Find the irreducible set from the following FDs

$$
\begin{aligned}
& \mathrm{F}= \\
& \mathrm{A} \rightarrow \mathrm{~B} \\
& \mathrm{C} \rightarrow \mathrm{~B} \\
& \mathrm{D} \rightarrow \mathrm{ABC} \\
& \mathrm{AC} \rightarrow \mathrm{D}
\end{aligned}
$$

## Sol:

## Step 1:

(1) $\mathrm{A} \rightarrow \mathrm{B}$
(2) $C \rightarrow B$
(3) $\mathrm{D} \rightarrow \mathrm{A}$
(4) $D \rightarrow B$
(5) $D \rightarrow C$
(6) $\mathrm{AC} \rightarrow \mathrm{D}$

## Step 2:

Remove 1 \& compute $\mathrm{A}^{+}$from $2,3,4,5,6$
$\mathrm{A}^{+}=\mathrm{A}$
$\therefore$ We need 1
Remove 2 and compute 1, 3, 4, 5\&6
$\mathrm{C}^{+}=\mathrm{C}$
$\therefore$ We need 2 .
Remove 3 and compute $\mathrm{D}^{+}$from 1, 2, 4, 5\&6
$\mathrm{D}^{+}=\mathrm{DBC}$
$\therefore$ We need 3 .
Remove 3 and compute $\mathrm{D}^{+}$from 1, 2, 4, 5\&6
$\mathrm{D}^{+}=\mathrm{DBC}$
$\therefore$ We need 3 .
Remove 4 and compute $\mathrm{D}^{+}$from 1, 2, 4, 5\&6
$\mathrm{D}^{+}=\mathrm{ADCB}$
$\therefore \mathrm{D} \rightarrow \mathrm{B}$ can be removed.
Remove 5 and compute $\mathrm{D}^{+}$from 1, 2, 3,4\&6
$\mathrm{D}^{+}=\mathrm{ABD}$
$\therefore$ We need 5 .
Remove 6 and compute $\mathrm{D}^{+}$from $1,2,3,4,5$
$\mathrm{AC}^{+}=\mathrm{ACB}$
$\therefore$ We need 6 .

## Step 3:

A $\rightarrow$ B
$\mathrm{C} \rightarrow \mathrm{B}$
$\mathrm{D} \rightarrow \mathrm{A}$
$\mathrm{D} \rightarrow \mathrm{C}$
$\mathrm{AC} \rightarrow \mathrm{D}$
Remove A
$\begin{aligned} & \mathrm{A} \rightarrow \mathrm{B} \\ & \mathrm{C} \rightarrow \mathrm{B} \\ & \mathrm{D} \rightarrow \mathrm{A} \\ & \mathrm{D} \rightarrow \mathrm{C} \\ & \mathrm{C} \rightarrow \mathrm{D} \\ & \mathrm{C}^{+}=\mathrm{CDAB}\end{aligned} \quad \begin{gathered}\mathrm{A} \rightarrow \mathrm{B} \\ \mathrm{C} \rightarrow \mathrm{B} \\ \mathrm{D} \rightarrow \mathrm{A} \\ \mathrm{D} \rightarrow \mathrm{C} \\ \mathrm{AC} \rightarrow \mathrm{D}\end{gathered} \mathrm{C}^{+}=\mathrm{CB}$
$\therefore \mathrm{C}^{+} \neq \mathrm{C}^{+}$

Remove C
$\mathrm{A} \rightarrow \mathrm{B}$
$\mathrm{C} \rightarrow \mathrm{B}$
$\mathrm{D} \rightarrow \mathrm{A}$

$\mathrm{D} \rightarrow \mathrm{C}$$\quad$| $\mathrm{A} \rightarrow \mathrm{B}$ |
| :--- |
| $\mathrm{C} \rightarrow \mathrm{B}$ |
| $\mathrm{A} \rightarrow \mathrm{D}$ |
|  |
|  |
| $\mathrm{A}^{+}=\mathrm{ADCB}$ |$\quad$| $\mathrm{D} \rightarrow \mathrm{A}$ |
| :--- |
| $\mathrm{AC} \rightarrow \mathrm{C}$ |
| $\mathrm{A}^{+}=\mathrm{AB}$ |

$\mathrm{A}^{+} \neq \mathrm{A}^{+}$

## Step 4:

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~B} \\
& \mathrm{C} \rightarrow \mathrm{~B} \\
& \mathrm{D} \rightarrow \mathrm{~A} \\
& \mathrm{D} \rightarrow \mathrm{C} \\
& \mathrm{AC} \rightarrow \mathrm{D} \\
& \\
& \mathrm{~A} \rightarrow \mathrm{~B} \\
& \mathrm{C} \rightarrow \mathrm{~B} \quad \text { Therefore, it is an irreducible F.D. } \\
& \mathrm{D} \rightarrow \mathrm{AC} \\
& \mathrm{AC} \rightarrow \mathrm{D}
\end{aligned}
$$

## Q2 Consider Universal relation with attributes ABC and FDs

$\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{C} \rightarrow \mathrm{B}$
$A \rightarrow B$
Find the Irreducible set
Sol:
Step 2:
Remove (1) \& compute $\mathrm{AB}^{+}$from $2 \& 3$
$\mathrm{AB}^{+}=\mathrm{AB}$
$\therefore$ We need 1
Remove (2) \& compute $\mathrm{AB}^{+}$from $1 \& 3$
$\mathrm{C}^{+}=\mathrm{c}$
$\therefore$ We need 2
Remove (3) \& compute $\mathrm{A}^{+}$from 1\&2
$\mathrm{A}^{+}=\mathrm{A}$
$\therefore$ We need 3
Step: 3
$\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{C} \rightarrow \mathrm{B}$
$A \rightarrow B$

Remove A
$\mathrm{AB} \rightarrow \mathrm{C} \quad \mathrm{B} \rightarrow \mathrm{C}$
$\mathrm{C} \rightarrow \mathrm{B} \quad \mathrm{C} \rightarrow \mathrm{B}$
$\mathrm{A} \rightarrow \mathrm{B} \quad \mathrm{A} \rightarrow \mathrm{B}$
$\mathrm{B}^{+}=\mathrm{B} \quad \mathrm{B}^{+}=\mathrm{BC}$
$\therefore \mathrm{B}^{+} \neq \mathrm{B}^{+}$

$\mathrm{A}^{+}=\mathrm{ABC} \quad \mathrm{A}^{+}=\mathrm{ACB}$
$\therefore \mathrm{A}^{+}=\mathrm{A}^{+}$
$\therefore$ B can be removed

## Step 4:

$A \rightarrow C$
$\mathrm{C} \rightarrow \mathrm{B}$
$A \rightarrow B$
$\therefore \mathrm{A} \rightarrow \mathrm{BC} \& \mathrm{C} \rightarrow \mathrm{B}$ it is an irreducible set.

## Q3 FDs are

$\mathrm{F}=\mathrm{ABD} \rightarrow \mathrm{AC}$

$$
\mathrm{C} \rightarrow \mathrm{BE}
$$

$\mathrm{AD} \rightarrow \mathrm{BF}$
$\mathrm{B} \rightarrow \mathrm{E}$
Find the minimal set

## Step 1:

$\mathrm{ABD} \rightarrow \mathrm{A}$
$\mathrm{ABD} \rightarrow \mathrm{C}$
$\mathrm{C} \rightarrow \mathrm{B}$
$\mathrm{C} \rightarrow \mathrm{E}$
$A D \rightarrow B$
$A D \rightarrow F$
$B \rightarrow E$
Remove (1) \& compute $\mathrm{ABD}^{+}$from (2-7)
$\mathrm{ABD}^{+}=\mathrm{ABDCEF}$
(1) can be removed

Remove (2) \& compute $\mathrm{ABD}^{+}$from (1,3-7)
$\mathrm{ABD}^{+}=\mathrm{ABDEF}$ $\therefore$ We need (2)

Remove (3) \& compute $\mathrm{C}^{+}$from (1,2,4-7)
$\mathrm{C}^{+}=\mathrm{CE}$
$\therefore$ We need (3)
Remove (4) \& compute $\mathrm{C}^{+} \mathrm{C}^{+}=\mathrm{BCE}$
(4) Can be removed

Remove (5) \& compute $\mathrm{AD}^{+}$
$\mathrm{AD}^{+}=\mathrm{ADF}$
$\therefore$ We need (5)
Remove (6) \& compute $\mathrm{AD}^{+}$
$\mathrm{AD}^{+}=\mathrm{ADBCE}$
$\therefore$ We need (6)
Remove (7) \& compute $\mathrm{B}^{+}$
$\mathrm{B}^{+}=\mathrm{B}$
$\therefore$ We need (7)

## Step 3:

ABD $\rightarrow \mathrm{C}$
$\mathrm{C} \rightarrow \mathrm{B}$
$A D \rightarrow B$
$\mathrm{AD} \rightarrow \mathrm{F}$
$B \rightarrow E$

|  | Remove A |
| :--- | :--- |
| $\mathrm{ABD} \rightarrow \mathrm{C}$ |  |
| $\mathrm{C} \rightarrow \mathrm{B}$ |  |
| $\mathrm{AD} \rightarrow \mathrm{B}$ |  |
| $\mathrm{AD} \rightarrow \mathrm{F}$ |  |$\quad$| $\mathrm{BD} \rightarrow \mathrm{C}$ |
| :--- |
| $\mathrm{C} \rightarrow \mathrm{B}$ |$\quad$| $\mathrm{AD} \rightarrow \mathrm{B}$ |
| :--- |
| $\mathrm{AD} \rightarrow \mathrm{F}$ |

$B \rightarrow E \quad B \rightarrow E$
$\mathrm{BD}^{+}=\mathrm{BDE} \quad \mathrm{BD}^{+}=\mathrm{BDCE}$
$\therefore \mathrm{BD}^{+} \neq \mathrm{BD}^{+}$
Remove B

| $\mathrm{ABD} \rightarrow \mathrm{C}$ | $\mathrm{AD} \rightarrow \mathrm{C}$ |
| :--- | ---: |
| $\mathrm{C} \rightarrow \mathrm{B}$ | $\mathrm{C} \rightarrow \mathrm{B}$ |
| $\mathrm{AD} \rightarrow \mathrm{B}$ | $\mathrm{AD} \rightarrow \mathrm{B}$ |
| $\mathrm{AD} \rightarrow \mathrm{F}$ | $\mathrm{AD} \rightarrow \mathrm{F}$ |
| $\mathrm{B} \rightarrow \mathrm{E}$ | $\mathrm{B} \rightarrow \mathrm{E}$ |

$\mathrm{AD}^{+}=\mathrm{ABFECD} \quad \mathrm{AD}^{+}=\mathrm{ADCFBE}$

$$
\therefore \mathrm{AD}^{+}=\mathrm{AD}^{+}
$$

$B$ can be removed.

## Types of functional Dependencies:

(1) Partial F.D
(2) Transitive F.D
(3) Full F.D.

1. Partial F.D: A dependency in which non-key attributes are partially depending on key attributes.
$\mathrm{R}=\mathrm{ABCD}$
$\mathrm{F}=\mathrm{AB} \rightarrow \mathrm{C}$
$=B \rightarrow D$
Key: $A B$ but $B$ is depending only $D$ therefore $B \rightarrow D$ is considered as partial dependency

## Under the following conditions a table cannot have partial F.D

(1) If primary key consists a single attribute
(2) If table consists only two attributes
(3) If all the attributes in the table are part of the primary key.
2. Transitive F.D: A dependency in which there is a relationship among the non-key attributes.

Eg:R=ABCD
F: $A B \rightarrow C$
$A B \rightarrow D$
$\mathrm{C} \rightarrow \mathrm{D}$
Key=AB
$\therefore \mathrm{C} \rightarrow \mathrm{d}$ Is a transitive dependency and it causes insertion, deletion $\&$ updation problems in the table.

## Under the following Circumstances, a table cannot have transitive F.D

(1) If table consists only two attributes
(2) If all the attributes in the table are part of the primary key.
3. Full F.D:A dependency $X \rightarrow Y$ is considered as a full $F$.D if the removal of any attribute from X makes $\mathrm{X} \rightarrow \mathrm{Y}$ as invalid F.D

Eg: $\mathrm{AB} \rightarrow \mathrm{CD}$
$B \rightarrow C D X$
$\therefore \mathrm{AB} \rightarrow \mathrm{CD}$ is a full F.D

## NORMALIZATION

$>$ It is the process of reducing the redundancy based on primary keys and F.D

## OR

> It is a tool to validate or evaluate the logical database design with the help of rules which are called as Normal Forms. They are

1 NF
2 NF
3 NF
BCNF increased.

Problem intensity reduces and no. of tables needed will be

4 NF
5 NF
DKNF

## Points to be Remember

$>1 \mathrm{NF}$ is a mandatory NF and remaining are the optional
$>$ If you construct E-R diagrams in to the tables, then 4 NF and 5 NF need not be applied on the table.
$>$ Practically applied normalization is upto 3NF and very rarely we will go beyond that.
$>2$ NF dealing with the partial dependencies and 3NF is dealing with transitive dependencies.

## First Normal Form (1NF):

$>$ The cells of the table must have single atomic value
$>$ Neither repeating groups nor arrays are allowed as values
$>$ All entries in any column (attribute)must be of the same kind
$>$ Each column must have a unique name, but the order of the columns in the table is insignificant.
$>$ No two rows in a table may be identical and the order of rows is insignificant

## Second Normal Form (2 NF):

$>$ A table is said to be in 2 NF if it is already in the 1 NF and free from Partial dependency.
> Anomalies can occur when attributes are dependent on only part of a multi-attribute key.

- A relation is in second normal form when all non-key attributes are dependent on the whole key.
$>$ Any relation having a key with a single attribute is in second normal form


## Q1 Consider universal relation $R=A B C D E F G H I J$ and the set of FDs are

$\mathrm{F}=\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{A} \rightarrow \mathrm{DE}$
$B \rightarrow F$
$\mathrm{F} \rightarrow \mathrm{GH}$
$\mathrm{D} \rightarrow \mathrm{IJ}$

What is the key for R? Decompose R into 2NF

Sol: key=AB
$\therefore \mathrm{AB}=$ CDEFGHIJ

$\mathrm{A}^{+}=\mathrm{ADEIJ}$
$\mathrm{B}^{+}=\mathrm{BFGH}$
If there is a partial dependency, remove partially dependent attributes from the original table and place it in a separate table along with the copy of its determinant.
(a) Key =AB
(b) $\mathrm{A}^{+}=\mathrm{ADEIJ}$
$\mathrm{B}^{+}=\mathrm{BFGH}$
DEIJ-depending only on A
FGH-depending only on B
$\mathrm{R}_{1}=\mathrm{ADEIJ}$
(c) $\left.\mathrm{R}_{2}=\mathrm{BFGH}\right\} \therefore \mathrm{R}=\mathrm{ABC}$
$\mathrm{R}_{3}=\mathrm{ABC}$
$\therefore$ Required 2 NF
Q2 Consider the relation $\mathrm{R}=\mathrm{ABCDEF}$ and set of FDs are
$\mathrm{F}=\mathrm{A} \rightarrow \mathrm{FC}$
$\mathrm{C} \rightarrow \mathrm{D}$
$B \rightarrow E$

## Find the key and normalize into 2NF

## Sol:

(a) $\mathrm{Key}=\mathrm{AB}$
(b) $\mathrm{A}^{+}=\mathrm{ACDF}$
$\mathrm{B}^{+}=\mathrm{BE}$
$\mathrm{R}_{1}=\mathrm{ACDF}$
(c) $\left.\begin{array}{rl}\mathrm{R}_{2} & =\mathrm{BE} \\ \mathrm{R}_{3} & =\mathrm{AB}\end{array}\right\} \therefore \mathrm{R}=\mathrm{AB}$
$\therefore$ Requried 2 NF

Q3 Consider the relation $\mathrm{R}=\mathrm{ABCDE}$. Find the key and normalize upto 2NF $\mathrm{F}=\mathrm{B} \rightarrow \mathrm{E}$
$\mathrm{C} \rightarrow \mathrm{D}$
$A \rightarrow B$
Sol:(A) KEY=AC
(B) $\mathrm{A}^{+}=\mathrm{ABE}$
$\mathrm{C}^{+}=\mathrm{CD}$
(C) $R=A$
$\left.\begin{array}{l}\mathrm{R}_{1}=\mathrm{ABE} \\ \mathrm{R}_{2}=\mathrm{CD} \\ \mathrm{R}_{3}=\mathrm{AC}\end{array}\right\} \therefore$ Required 2 NF

## Q4 Consider R=ABCDEFGHIJ and FDs are

$\mathrm{F}=\mathrm{AB} \rightarrow \mathrm{C}$

$$
\mathrm{BD} \rightarrow \mathrm{EF}
$$

$\mathrm{AD} \rightarrow \mathrm{GH}$
$\mathrm{A} \rightarrow \mathrm{I}$
$\mathrm{H} \rightarrow \mathrm{J}$
Find the key and normalize upto 2 NF

Sol: (A) Key=ABD
(B) $\mathrm{AB}^{+}=\mathrm{ABCI}$
$\mathrm{BD}^{+}=\mathrm{BDEF}$
$\mathrm{AD}^{+}=\mathrm{ADGHIJ}$
$(\mathrm{C}) \mathrm{R}_{1}=\mathrm{ABCI} \triangle \begin{array}{ll}\mathrm{A}^{+}=\mathrm{AI} \longrightarrow & \mathrm{R}_{1}{ }^{1}=\mathrm{AI} \\ \mathrm{B}^{+}=\mathrm{B} & \mathrm{R}_{1}{ }^{11}=\mathrm{ABC}\end{array}$
$\mathrm{R}_{2}=\mathrm{BDEF} \quad \mathrm{B}^{+}=\mathrm{B} \quad \mathrm{R}_{2}=\mathrm{BDEF}$
$\begin{aligned} & \mathrm{R}_{3}=\mathrm{ADGHH} \quad \mathrm{A}^{+}=\mathrm{AI}-\mathrm{R}_{3}{ }^{1}=\mathrm{AI} \\ & \mathrm{D}^{+}=\mathrm{D}-\mathrm{R}_{3}{ }^{11}=\mathrm{ADGHJ}\end{aligned}$

$$
\mathrm{R}_{4}=\mathrm{ABD}
$$

Third Normal Form (3 NF): A table is said to be in the 3 NF is it is already in the 2 NF and must be free from transitive dependencies.
$>$ Anomalies can occur when a relation contains one or more transitive
$>$ A transitive dependency exists when $\mathrm{A} \rightarrow \mathrm{B} \rightarrow \mathrm{C}$ and NOT $\mathrm{B} \rightarrow \mathrm{A}$
$>$ A relation is in 3NF when it is in 2 NF and has no transitive dependency
> A relation is in 3NF when "All non-key attributes are dependent on the key, the whole key and nothing but the key".

If there is a transitive dependency, remove transitively dependent attribute from 2 NF table and place it in a separate table along with the copy of its determinant.

## Update anomalies occur in an 3NF relation $R$ if

> R has multiple candidate keys,
> Those candidate keys are composite, and
> The candidate keys are overlapped

## Q1 Consider universal relation $R=A B C D E F G H I J$ and the set of FDs are

$\mathrm{F}=\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{A} \rightarrow \mathrm{DE}$
$B \rightarrow F$
$\mathrm{F} \rightarrow \mathrm{GH}$
$\mathrm{D} \rightarrow \mathrm{IJ}$
Decompose R into 3NF

Transitive $\mathrm{D} \rightarrow \mathrm{IJ}$ ADEIJ
$\therefore$ DIJ
ADE


## BP

$\mathrm{ABC} \rightarrow \underline{\mathrm{ABC}}$
$\left.\begin{array}{ll}\mathrm{R}_{1}=\mathrm{DIJ} & \mathrm{R}_{1}=\mathrm{DIJ} \\ \mathrm{R}_{2}=\mathrm{ADE} & \mathrm{R}_{2}=\mathrm{ADE} \\ \mathrm{R}_{3}=\mathrm{FGH} & \mathrm{R}_{3}=\mathrm{FGH} \\ \mathrm{R}_{4}=B F & \mathrm{R}_{4}=\mathrm{BF} \\ \mathrm{R}_{5}=A B C & \mathrm{R}_{5}=\mathrm{ABC}\end{array}\right\}$ Iti sin 3 NF

Q2 Consider the relation $\mathrm{R}=\mathrm{ABCDEF}$ and set of FDs are
$\mathrm{F}=\mathrm{A} \rightarrow \mathrm{FC}$

$$
\mathrm{C} \rightarrow \mathrm{D}
$$

$B \rightarrow E$

## Normalize $R$ into 3NF

$\mathrm{ACDF}=\mathrm{CD}$
$\mathrm{BE} \rightarrow \underline{\mathrm{BE}}$
$A B \rightarrow \underline{\mathrm{AB}}$
$\left.\begin{array}{l}\mathrm{R}_{1}=\mathrm{CD} \\ \mathrm{R}_{2}=\mathrm{ACF} \\ \mathrm{R}_{3}=\mathrm{BE} \\ \mathrm{R}_{4}=A B\end{array}\right\}$ in $3 N F$

Q3 Consider the relation $\mathrm{R}=\mathrm{ABCDE}$. Find the key and normalize upto 3NF $\mathrm{F}=\mathrm{B} \rightarrow \mathrm{E}$
$\mathrm{C} \rightarrow \mathrm{D}$
$A \rightarrow B$
$\xrightarrow{\mathrm{R}_{1}=\mathrm{ABE} \underset{\mathrm{AB}}{\longrightarrow} \mathrm{BE} \text {. }}$
$\left.\begin{array}{l}R_{2}=\mathrm{CD} \\ \mathrm{R}_{3}=\mathrm{AC} \quad \mathrm{AC} \\ \mathrm{R}_{1}=\mathrm{BE} \\ \mathrm{R}_{2}=\mathrm{AB} \\ \mathrm{R}_{3}=\mathrm{CD} \\ \mathrm{R}_{4}=\mathrm{AC}\end{array}\right\}$ Iti $\sin 3 \mathrm{NF}$

## Q4 Consider $\mathrm{R}=$ ABCDEFGHIJ and FDs are

$\mathrm{F}=\mathrm{AB} \rightarrow \mathrm{C}$

$$
\mathrm{BD} \rightarrow \mathrm{EF}
$$

$$
\mathrm{AD} \rightarrow \mathrm{GH}
$$

$\mathrm{A} \rightarrow \mathrm{I}$

$$
\mathrm{H} \rightarrow \mathrm{~J}
$$

Normalize upto 3NF
$\mathrm{R}_{1}=\mathrm{AI}$
$\mathrm{R}_{2}=\mathrm{ABC}$
$\mathrm{R}_{3}=\mathrm{BDEF}$
$\mathrm{R}_{4}=\mathrm{ADGH} \underset{\mathrm{ADG}}{\mathrm{HJ}}$
$\mathrm{R}_{5}=\mathrm{ABD}$
$\left.\begin{array}{l}\mathrm{R}_{1}=\mathrm{AI} \\ \mathrm{R}_{2}=\mathrm{ABC} \\ \mathrm{R}_{3}=\mathrm{BDEF} \\ \mathrm{R}_{4}=\mathrm{HJ} \\ \mathrm{R}_{5}=\mathrm{ADG} \\ \mathrm{R}_{6}=\mathrm{ABD}\end{array}\right\}$ Iti $\sin 3 \mathrm{NF}$
Q5(a) Give a set of FDs for the relation schema $R(A B C D)$ with primary
key AB under which $R$ is 1NF but not in 2NF
(b) Find FDs such that $R$ is in 2NF but not in 3NF

R=ABCD
Key=AB
Sol: (a) with these FD's table cannot be in 2NF

$$
\mathrm{B} \rightarrow \mathrm{C} \quad \mathrm{~A} \rightarrow \mathrm{C}
$$

$\mathrm{B} \rightarrow \mathrm{D} \quad \mathrm{A} \rightarrow \mathrm{D}$
(b) with these FD's the table may be in 2NF but not in 3NF
$\mathrm{C} \rightarrow \mathrm{D} \quad \mathrm{D} \rightarrow \mathrm{C}$
Note 1: In general if $\mathrm{x} \rightarrow \alpha$; if $\alpha \neq$ AorB and $\mathrm{x}=\mathrm{A}$ or $\mathrm{x}=\mathrm{b}(\mathrm{key}=\mathrm{AB})$ then it will violate 2 NF

Note 2: In general, if I have $\mathrm{x} \rightarrow \alpha$; with $\alpha \neq \mathrm{A}$ or $\alpha \neq \mathrm{B}$ and x is not a proper set of AB then it will violet 3NF but not 2 NF .

Note 3: If there is a F.D $x \rightarrow y$, It is allowed in 3NF(also in $2 N F$ ) if $x$ is a super key or y is a part of key.

## Lossless join property and dependency preserving:

In a good database design, it is not only sufficient to check whether tables are satisfying $2 \mathrm{NF}, 3 \mathrm{NF}$ \& BCNF but also check whether decomposed tables are satisfying the following two properties.
(1) Lossless join Property(mandatory)
(2) Dependency Preserving Property(optional)

## Lossless join Property:

A decomposition of R into $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}, \ldots . . \mathrm{R}_{\mathrm{n}}$ is said to a loss less decomposition if the natural joint of all the projections of R must give the original relation i.e $R=\Pi_{R 1}(R) \propto \Pi_{R 2}(R) \ldots . . . \Pi_{R n}(R)$ suppose, if $\mathrm{R} \subseteq \Pi_{\mathrm{R} 1}(\mathrm{R}) \propto \Pi_{\mathrm{R} 2}(\mathrm{R}) \ldots \ldots . . \Pi_{\mathrm{Rn}}(\mathrm{R})$ then decomposition is said to be lossy decomposition.

## Q:

R:

| A | B | C |
| :--- | :--- | :--- |
| $\mathrm{a}_{1}$ | $\mathrm{~b}_{1}$ | $\mathrm{c}_{1}$ |
| $\mathrm{a}_{2}$ | $\mathrm{~b}_{2}$ | $\mathrm{c}_{2}$ |


| $\mathrm{a}_{3}$ | $\mathrm{~b}_{1}$ | $\mathrm{c}_{3}$ |
| :--- | :--- | :--- |

$\mathrm{R}_{1}$
$\mathrm{R}_{2}$

| A | B |
| :--- | :--- |
| $\mathrm{a}_{1}$ | $\mathrm{~b}_{1}$ |
| $\mathrm{a}_{2}$ | $\mathrm{~b}_{2}$ |
| $\mathrm{a}_{3}$ | $\mathrm{~b}_{1}$ |


$\therefore \mathrm{R}=\Pi_{\mathrm{R} 1}(\mathrm{R}) \infty \Pi_{\mathrm{R} 2}(\mathrm{R})$

| A | B | C |
| :--- | :--- | :--- |
| $\mathrm{a}_{1}$ | $\mathrm{~b}_{1}$ | $\mathrm{c}_{1}$ |
| $\mathrm{a}_{1}$ | $\mathrm{~b}_{1}$ | $\mathrm{c}_{3}$ |
| $\mathrm{a}_{2}$ | $\mathrm{~b}_{2}$ | $\mathrm{c}_{2}$ |
| $\mathrm{a}_{3}$ | $\mathrm{~b}_{1}$ | $\mathrm{c}_{1}$ |
| $\mathrm{a}_{3}$ | $\mathrm{~b}_{1}$ | $\mathrm{c}_{3}$ |

$\therefore 5$ rows
$\therefore$ Lossy decomposition

The above method is a time consuming and error prone there fore,to check the lossless joint property we use the following short cut method.
i.e, If the common column $b / w$ the relation consists unique value(only primary key willwill have unique values) then the decomposition will become lossless otherwise it is a lossy decomposition.

$$
\begin{gathered}
\mathrm{R}_{1} \cap \mathrm{R}_{2} \rightarrow \mathrm{R}_{1} \\
\text { or } \\
\mathrm{R}_{1} \cap \mathrm{R}_{2} \rightarrow \mathrm{R}_{2}
\end{gathered}
$$

## Dependency preserving property:

Decomposition of R into $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}, \ldots . . \mathrm{R}_{\mathrm{n}}$ is said to be a dependency preserving decomposition if, $\left(\mathrm{F}_{1,}, \mathrm{~F}_{2} \ldots \ldots . . . \mathrm{F}_{\mathrm{n}}\right)^{+}=\mathrm{F}^{+}$where $\mathrm{F}_{1}$ is the set of F.D's in $R_{1}, F_{2}$ is the set of F.D's in $R_{2}$ and so on and $F$ is the set of F.D's in R.

Eg:consider a table consists $R=A B C D$ attributes then $F=A \rightarrow B, A \rightarrow C, C \rightarrow D$ is decomposed into $R_{1}(A B C), R_{2}(C D)$.find whether this decomposition is satisfying lossless join and dependency preserving property.

Sol: $\quad R_{1}(A B C)$
$\mathrm{R}_{2}$ (CD)

It is a lossless decomposition and dependency preserving relation.
$A \rightarrow B\left(R_{1}\right)$
$A \rightarrow C\left(R_{1}\right)$
$\mathrm{C} \rightarrow \mathrm{D}\left(\mathrm{R}_{2}\right)$

If the table is decomposed like $R_{1}(A B D), R_{2}(B C)$ it is a lossy decomposition $\therefore$ It is also not a dependency preserving relation.

## Boyce-Codd Normal Form (BCNF):

A table is said to be in BCNF, if it is already in the 3NF and if every non trivial F.D has a candidate key as its determinant.

A table is said to be in BCNF if all determinants are keys in the 3NF table or they must be super keys
$>$ Anomalies can occur in relation in 3NF if there are determinants in the relation that are not candidate keys.
$>$ A relation is in BCNF if every determinant is a candidate key,
$>$ To test whether a relation is in BCNF, we identify all the determinants and make sure that they are candidate key.

The following conditions are not properly handled by 3NF
(1) If table consists two or more composite primary keys
(2) If the candidate keys overlap.

## 3 NF

1.It concentrates on the primary key
2.Redundancy is high compared to BCNF
3.It may preserve all dependencies
4.If there is a dependency $x \rightarrow y$ is allowed in 3NF if x is a super key or $Y$ is the part of key

## 2 NF

it concentrates on all candidate keys

Redundancy is low compared to 3 NF

It may not preserve all F.D's If there is a dependency $x \rightarrow$ y.It is allowed in BCNF if X is super key.

Q:
(1) $R=A B C D$
$\mathrm{A} \rightarrow \mathrm{D}$
$\mathrm{C} \rightarrow \mathrm{A}$
$B \rightarrow C$

## Sol:

(a) $K e y=b$
(b) At present $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ one in 2 NF
(c) BACD

## (2) $\mathrm{B} \rightarrow \mathrm{C}$

$\mathrm{D} \rightarrow \mathrm{A}$

## Sol:

(a) $\mathrm{Key}=\mathrm{BD}$
(b) 1 NF
(c) $2 \mathrm{NF}=\mathrm{BC}, \mathrm{DA}, \mathrm{BD}$
$3 \mathrm{NF}=\mathrm{BC}, \mathrm{DA}, \mathrm{BD}$
(d) $\mathrm{BCNF}=\mathrm{BC}, \mathrm{DA}, \mathrm{BD}$
$B C D \cap D A$
$\therefore$ Lossless decomposition.
(3) $\mathrm{ABC} \rightarrow \mathrm{D}$
$\mathrm{D} \rightarrow \mathrm{A}$

## Sol:

(a) $\mathrm{Key}=\mathrm{ABC}, \mathrm{BCD}$
(b) let key=ABC
$\therefore 3 \mathrm{NF}$
(c) $\mathrm{BCNF}=\mathrm{ABC}$

Super key ABC $\rightarrow$ D
$\mathrm{D} \rightarrow \mathrm{A}$
$\therefore \mathrm{BCD}$, DA (but ABC will not be a key)
Q. $R=A B C D$

$$
\begin{gathered}
A \rightarrow B \\
B C \rightarrow D
\end{gathered}
$$

$$
A \rightarrow C
$$

## Sol:

a). $k e y=A$
b). 2 NF
c). $3 \mathrm{NF}=\mathrm{BCD}, \mathrm{ABC}$
d). BCNF:

$$
\begin{array}{cc}
\text { A } & \\
& \Rightarrow \quad \mathrm{BCNF}
\end{array}
$$

BC BC
Q). $\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{AB} \rightarrow \mathrm{D}$
$\mathrm{C} \rightarrow \mathrm{A}$
$\mathrm{D} \rightarrow \mathrm{B}$

Sol:
a). $k e y=A B, B C, C D, A D$
B). CHOOSE AB as a key then <ABCD>
$\therefore 3 N F$
c). AB AB

C $\mathrm{C} \rightarrow \mathrm{AX}$

D $\mathrm{D} \rightarrow \mathrm{BX}$
$\therefore \mathrm{CA}, \mathrm{DB}, \mathrm{CD}($ but AB absent)
$\therefore \mathrm{AB}, \mathrm{CD}, \mathrm{CA}, \mathrm{DB}($ add AB$)$
OR

Sol is $3 N F$
Q). $\mathrm{AB} \rightarrow \mathrm{CEFG}$
$\mathrm{A} \rightarrow \mathrm{D}$
$\mathrm{F} \rightarrow \mathrm{G}$
$\mathrm{FB} \rightarrow \mathrm{H}$
$\mathrm{HBC} \rightarrow \mathrm{ADEFG}$
$\mathrm{FBC} \rightarrow \mathrm{ADE}$

## Sol:

a). key $=\mathrm{AB}, \mathrm{HBC}, \mathrm{FBC}$
B). Choose key AB <ABCDEFGH> AD

ABCEFGH
$\therefore 2 \mathrm{NF}=\mathrm{AD}, \mathrm{ABCEFGH}$
c). $3 \mathrm{NF}=\mathrm{AD}, \mathrm{ABCEFGH} \mathrm{FG}$

ABCEFH

D). BCNF

AB A
A F
F $\quad \mathrm{AB}$
FB
HBC
FBC
$\therefore \mathrm{BCNF}=\mathrm{FG}, \mathrm{AD}, \mathrm{FBH}, \mathrm{ABCEF}$
Q). $R=A B C D$

$$
\mathrm{R}_{1}=\mathrm{BC} \& \mathrm{AD}\left(\mathrm{R}_{2}\right)
$$

(1).

$$
\begin{aligned}
& \mathrm{B} \rightarrow \mathrm{C} \\
& \mathrm{D} \rightarrow \mathrm{~A}
\end{aligned}
$$

## Sol:

(1) a).key=BD
b). $\mathrm{R}_{1} \cap \mathrm{R}_{2}=0$
$\therefore$ lossy decomposition
$\Rightarrow$ bad decomposition

The decomposition is bad decomposition
$\because$ it is not satisfying lossless join property but it is satisfying dependency preserving property.
(2). $\mathrm{AB} \rightarrow \mathrm{C}$
$\mathrm{C} \rightarrow \mathrm{A} \quad \mathrm{R}_{1}=\mathrm{ACD}$
$\mathrm{C} \rightarrow \mathrm{D} \quad \mathrm{R}_{2}=\mathrm{BC}$
A). $k e y=A B, B C$
b). $\mathrm{R}_{1} \cap \mathrm{R}_{2}=\mathrm{C}$
it is a lossless decomposition as common attribute ' $c$ ' can become a key for the first table ACD.
(3). $\mathrm{A} \rightarrow \mathrm{BC} \quad \mathrm{R}_{1}=\mathrm{ABC}$
$\mathrm{C} \rightarrow \mathrm{AD} \quad \mathrm{R}_{2}=\mathrm{AD}$
A). $\mathrm{KEY}=\mathrm{A}$
b). $\mathrm{R}_{1} \cap \mathrm{R}_{2}=\mathrm{A}$
$\therefore$ It is a lossless decomposition as A is a common attribute and can become a key.
(4). $A \rightarrow B$
$\mathrm{B} \rightarrow \mathrm{C}$
$\mathrm{R}_{1}=\mathrm{AB}$

$$
\mathrm{C} \rightarrow \mathrm{D} \quad \mathrm{R}_{2}=\mathrm{ACD}
$$

## Sol:

a). $k e y=A$
b). $\mathrm{R}_{1} \cap \mathrm{R}_{2}=\mathrm{A}$
$\therefore$ Lossless
FD not preserved $\because \mathrm{B} \rightarrow \mathrm{C}$
Q).

| $\mathrm{A} \rightarrow \mathrm{B}$ | $\mathrm{R}_{1}=\mathrm{AB}$ |
| :--- | ---: |
| $\mathrm{B} \rightarrow \mathrm{C}$ | $\mathrm{R}_{2}=\mathrm{AD}$ |
| $\mathrm{C} \rightarrow \mathrm{D}$ | $\mathrm{R}_{3}=\mathrm{CD}$ |

## Sol:

$\therefore$ It is a lossy decomposition
Q).
$\mathrm{R}=\mathrm{ABCDE}$
$\mathrm{AB} \rightarrow \mathrm{DE}$
$\mathrm{A} \rightarrow \mathrm{C}$
$\mathrm{D} \rightarrow \mathrm{E}$

## Sol:

(a) $\mathrm{Key}=\mathrm{AB}$
(b) $2 \mathrm{NF}=\mathrm{ABDE}, \mathrm{AC}$
(c) $3 \mathrm{NF}=\mathrm{ABD}, \mathrm{DE}, \mathrm{AC}$
(d) $B C N F=A B \quad A B$

D A
D A
Q)
$\mathrm{AB} \rightarrow \mathrm{CDE}$
$\mathrm{C} \rightarrow \mathrm{A}$

D $\rightarrow E$

## Sol:

(a) $\mathrm{Key}=\mathrm{AB}, \mathrm{BC}$
(b) choose AB as a key and no partial dependencies

3 NF=ABCD, DE
BCNF= ABCD,CA,DE

## Limitations of Normalization:

(1) It cannot detect redundancy among the tables.
(2) It may slow down the query retrival process
(3) The decomposed tables may not have real world meaning i.e, we cannot understand the significant of the tables straight away.

## Solutions:

$>$ To avoid all the problems selectively we go for De-Normalization i.e, decomposed relations are combined together to improve the performance of the speed up the data retrieval.

## Advantages:

(1) Database consistency can be achieved
(2) It increases the speed (By eliminating the duplicates)
(3) It reduces the disk size
(4) It maintains the integrity.

## Multi-Valued Dependencies (MVD)

The possible existence of multi-valued dependencies in a relation is due to first normal form (1NF), which disallows an attribute in a tuple from having a set of values.

For example, if we have two multi-valued attributes in a relation, we have to repeat each value of one of the attributes with every value of the other attribute, to ensure that tuples of a relation are consistent. This type of constraint is referred to as a multi-valued dependency and results in data redundancy.

Consider the Employee relation which is not in 1NF shown in figure:

| EmpNum | EmpPhone | EmpDegrees |
| :---: | :--- | :--- |
| 111 | $\{040-222222$, |  |
| $040-333333\}$ | $\{\mathrm{BA}, \mathrm{BSc}\}$ |  |

The result of 1 NF on above relation is shown below:

| EmpNum | EmpPhone | EmpDegrees |
| :--- | :--- | :--- |
| 111 | $040-222222$ | BA |
| 111 | $040-333333$ | BSc |
| 111 | $040-222222$ | BSc |
| 111 | $040-333333$ | BA |

This relation records the EmpPhone and EmpDegrees details of an employee 111. However, the EmpDegrees of an employee are independent of EmpPhone. This constraint results in data redundancy and is referred to as multi-valued dependency.

## Multi-Valued Dependency (MVD):

Represents a dependency between attributes (for example, A, B, and C) in a relation, such that for each value of $A$ there is a set of values for $B$ and a set of values for $C$. However, the set of values for $B$ and $C$ are independent of each other.

We represent an MVD between attributes $\mathrm{A}, \mathrm{B}$, and C in a relation using the following notation:
$\mathrm{A} \rightarrow \mathrm{B}$
$\mathrm{A} \rightarrow \mathrm{C}$

For example, we specify the MVD in the above Employee relation as follows:

EmpNum $\rightarrow$ EmpPhone

EmpNum $\rightarrow$ EmpDegrees

Formally, an MVD can be defined as:
Let $R$ be a relation schema, and $X$ and $Y$ be disjoint subsets of $R$, and $Z=$ $\mathrm{R}-\mathrm{XY}$.

If a relation R satisfies $\mathrm{X} \rightarrow \rightarrow \mathrm{Y}^{2}$, the following must be true for every legal instance of $r$ of $R$ :
if for any two tuples t 1 , t 2 and $\mathrm{t} 1(\mathrm{X})=\mathrm{t} 2(\mathrm{X})$, then there exist t 3 in $r$ such that
$\mathrm{t} 3(\mathrm{X})=\mathrm{t} 1(\mathrm{X}), \mathrm{t} 3(\mathrm{Y})=\mathrm{t} 1(\mathrm{Y}), \mathrm{t} 3(\mathrm{Z})=\mathrm{t} 2(\mathrm{Z})$.

By symmetry, there exist t 4 in $r$ such that, $\mathrm{t} 4(\mathrm{X})=\mathrm{t} 1(\mathrm{X}), \mathrm{t} 4(\mathrm{Y})=\mathrm{t} 2(\mathrm{Y}), \mathrm{t} 4(\mathrm{Z})=$ t1 (Z).

[^1]| $X$ | $Y$ | $Z$ |  |
| :--- | :--- | :--- | :--- |
| $x 1$ | $y 1$ | $z 1$ | $t 1$ |
| $x 1$ | $y 2$ | $z 2$ | $t 2$ |
| $x 1$ | $y 1$ | $z 2$ | $t 3$ |
| $x 1$ | $y 2$ | $z 1$ | $t 4$ |

The MVD $\mathrm{X} \rightarrow \rightarrow \mathrm{Y}$ says that the relationship between X and Y is independent of the relationship between X and $\mathrm{R}-\mathrm{Y}$.

## Armstrong Axioms rules relate to MVDs:

- MVD Complementation: If $X \rightarrow \rightarrow Y$, then $X \rightarrow \rightarrow R-X Y$.
- MVD Augmentation: If $X \rightarrow \rightarrow Y$ and $W$ be super set of $Z$, then $\mathrm{WX} \rightarrow \rightarrow \mathrm{YZ}$.
- MVD Transitivity: If $\mathrm{X} \rightarrow \rightarrow \mathrm{Y}$ and $\mathrm{Y} \rightarrow \rightarrow \mathrm{Z}$, then $\mathrm{X} \rightarrow \rightarrow(\mathrm{Z}-\mathrm{Y})$.


## Trivial MVD:

An MVD $\mathrm{A} \rightarrow \rightarrow \mathrm{B}$ in relation R is defined as being trivial if,
a) $B$ is a subset of $A$ or
b) $A \cup B=R$.

An MVD is said to be non-trivial if neither a) nor b) is satisfied.

## Fourth Normal Form (4NF):

A relation that is in Boyce-Codd normal form and contains no non-trivial MVDs.

Formally, Let R be a relation schema, X and Y be nonempty subsets of the attributes of $R . R$ is said to be in 4NF, if, for every MVD $X \rightarrow \rightarrow Y$ that holds over R , one of the following is true:

- $Y$ is a subset of $X$ or $X Y=R$, or
- $X$ is a super key.


## Example:

Consider the above Employee relation.
Identifying non-trivial MVD:
The Employee is not in 4NF because of the presence of non-trivial MVD.

Transformation into 4NF:

We decompose the Employee relation into Emp1 and Emp2 relations as shown below:

Emp1
Emp2

| EmpNum | EmpPhone |
| :--- | :--- |
| 111 | $040-$ <br> 222222 |
| 111 | $040-$ <br> 333333 |


| EmpNum | EmpDegrees |
| :--- | :--- |
| 111 | BA |
| 111 | BSc |

Both new relations are in 4NF because the Emp1 relation contains the trivial MVD EmpNum $\rightarrow$ EmpPhone, and the Emp2 relation contains the trivial MVD EmpNum $\rightarrow$ EmpDegrees.

## Properties of Decomposition

## Lossless-Join Decomposition:

Let $R$ be a relation schema and let $F$ be a set of $F D s$ over $R$. A decomposition of $R$ into two schemas with attributesets $X$ and $Y$ is said to be a losslessjoin decomposition with respect to $F$ if, for every instance $r$ of $R$ that satisfies the dependencies in $F, \Pi_{\mathrm{x}}(\mathrm{r})$ Пу (r) = r. In other words, we can recover the original relation from the decomposed relations.

From the definition it is easy to see that $r$ is always a subset of natural join of decomposed relations. If we take projections of a relation and recombine them using natural join, we typically obtain some tuples that were not in the original relation.

## Example:

By replacing the instance $r$ shown in figure with the instances $\prod_{\mathrm{sp}}(\mathrm{r})$ and $\prod_{\text {PD }}(\mathrm{r})$, we lose some information.

| S | P | D |
| :--- | :--- | :--- |
| s1 | p 1 | d 1 |
| s 2 | p 2 | d 2 |
| s 3 | p 1 | d 3 |


$\prod_{S P}(\mathrm{r})$

| $P$ | $D$ |
| :--- | :--- |
| p1 | d1 |
| p2 | d2 |
| p1 | d3 |

$П_{\mathrm{PD}}(\mathrm{r})$

| $S$ | $P$ | $D$ |
| :--- | :--- | :--- |


| s1 | p1 | d1 | $\prod_{\mathrm{sp}}$ |
| :---: | :---: | :---: | :---: |
| s2 | p2 | d2 |  |
| s3 | p1 | d3 |  |
| s1 | p1 | d3 |  |
| s3 | p1 | d1 |  |

Fig: Instances illustrating Lossy Decompositions

Theorem: Let R be a relation and F be a set of FDs that hold over R . The decomposition of R into relations with attribute sets R 1 and R 2 is lossless if and only if $\mathrm{F}^{+}$contains either the FD R1 $\cap \mathrm{R} 2 \rightarrow \mathrm{R} 1$ (or $\mathrm{R} 1-\mathrm{R} 2$ ) or the FD R1 $\cap \mathrm{R} 2 \rightarrow \mathrm{R} 2$ (or R2-R1).

Consider the Hourly_Emps relation. It has attributes $S N L R W H$, and the FD $\mathrm{R} \rightarrow \mathrm{W}$ causes a violation of 3NF. We dealt this violation by decomposing the relation into $S N L R H$ and $R W$. Since R is common to both decomposed relations and $\mathrm{R} \rightarrow \mathrm{W}$ holds, this decomposition is lossless-join.

## Dependency-Preserving Decomposition:

Consider the Contracts relation with attributes $C S J D P Q V$. The given FDs are $\mathrm{C} \rightarrow \mathrm{CSJDPQV}, \mathrm{JP} \rightarrow \mathrm{C}$, and $\mathrm{SD} \rightarrow \mathrm{P}$. Because SD is not a key, the dependency $\mathrm{SD} \rightarrow \mathrm{P}$ causes a violation of BCNF.

We can decompose Contracts into relations with schemas CSJDQV and SDP to address this violation. The decomposition is lossless-join. But, there is one problem. If we want to enforce an integrity constraint $\mathrm{JP} \rightarrow \mathrm{C}$, it requires an expensive join of the two relations. We say that this decomposition is not dependency-preserving.

Let R be a relation schema that is decomposed into two schemas with attributes sets X and Y , and let F be a set of FD over R . The projection of $\mathbf{F}$ on $\mathbf{X}$ is the set of FDs in the closure $\mathrm{F}^{+}$that involve only attributes in X . We denote the projection of F on attributes X as $\mathrm{F}_{\mathrm{X}}$. Note that a dependency $\mathrm{U} \rightarrow \mathrm{V}$ in $\mathrm{F}^{+}$is in $\mathrm{F}_{\mathrm{x}}$ only if all the attributes in U and V are in X .

The decomposition of relation schema R with FDs F into schemas with attribute sets X and Y is dependency-preserving if $\left(\mathrm{F}_{\mathrm{X}} \mathrm{U} \mathrm{F}_{\mathrm{Y}}\right)^{+}=\mathrm{F}^{+}$.

## Example:

Consider the relation $R$ with attributes $A B C$ is decomposed into relations with attributes $A B$ and $B C$. The set of FDs over $R$ includes $A \rightarrow B, B \rightarrow C$, and $\mathrm{C} \rightarrow \mathrm{A}$.

The closure of F contains all dependencies in F plus $\mathrm{A} \rightarrow \mathrm{C}, \mathrm{B} \rightarrow \mathrm{A}$, and $\mathrm{C} \rightarrow \mathrm{B}$. Consequently $\mathrm{F}_{\mathrm{AB}}$ contains
$A \rightarrow B$ and $B \rightarrow A$, and $F_{B C}$ contains $B \rightarrow C$ and $C \rightarrow B$. Therefore, $F_{A B} U F_{B C}$ contains $\mathrm{A} \rightarrow \mathrm{B}, \mathrm{B} \rightarrow \mathrm{C}, \mathrm{B} \rightarrow \mathrm{A}$
and $\mathrm{C} \rightarrow \mathrm{B}$. The closure of $\mathrm{F}_{\mathrm{AB}}$ and $\mathrm{F}_{\mathrm{BC}}$ now includes $\mathrm{C} \rightarrow \mathrm{A}$ (which follows from $\mathrm{C} \rightarrow \mathrm{B}$ and $\mathrm{B} \rightarrow \mathrm{A}$ ). Thus
the decomposition preserves the dependency $\mathrm{C} \rightarrow \mathrm{A}$.


[^0]:    ${ }^{1} X \rightarrow Y$ is read as $X$ functionally determines $Y$, or simply as $X$ determines $Y$.

[^1]:    ${ }^{2} X \rightarrow \rightarrow Y$ can be read as $X$ multi-determines $Y$.

